Weak and Strong Convergence Theorems for Krasnoselskij Iterative algorithm in the class of enriched strictly pseudocontractive and enriched nonexpansive operators in Hilbert Spaces

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Contents

- In this paper, we present some results about the aproximation of fixed points of enriched strictly pseudocontractive and enriched nonexpansive operators. There are numerous works in this regard (for example [9], [10], [11] [14], [16], [35] and references to them).

Introduction and Preliminaries

- In this paper, we present some results about the aproximation of fixed points of enriched strictly pseudocontractive and enriched nonexpansive operators. There are numerous works in this regard (for example [9], [10], [11] [14], [16], [35] and references to them).

- In order to approximate the fixed points of enriched strictly pseudocontractive and enriched nonexpansive mappings, we use the Krasnoselskij iterative algorithm for which we prove weak and strong convergence theorems.

Introduction and Preliminaries

- In this paper, we present some results about the aproximation of fixed points of enriched strictly pseudocontractive and enriched nonexpansive operators. There are numerous works in this regard (for example [9], [10], [11] [14], [16], [35] and references to them).

- In order to approximate the fixed points of enriched strictly pseudocontractive and enriched nonexpansive mappings, we use the Krasnoselskij iterative algorithm for which we prove weak and strong convergence theorems.

- Also, in this paper, we make a comparative study about some classical convergence theorems from the literature in the class of enriched strictly pseudocontractive and enriched nonexpansive mappings.

Definition 1 [9]

Let K be a nonempty subset of a real normed linear space X. A mapping $T : K \to K$ is called:

1. *nonexpansive* if
$$\|Tx - Ty\| \leq \|x - y\| \;\; \forall x, y \in K. \tag{1}$$
2. *pseudocontractive* if
$$\|Tx - Ty\|^2 \leq \|x - y\|^2 + \|x - y - (Tx - Ty)\| \;\; \forall x, y \in K. \tag{2}$$
3. *strictly pseudocontractive* if there exist $k < 1$ such that
$$\|Tx - Ty\|^2 \leq \|x - y\|^2 + k\|x - y - (Tx - Ty)\| \; \forall x, y \in K. \tag{3}$$

   In this case, T is also called *k-strictly pseudocontractive*.

Remark 1

Any strictly pseudocontractive operator is Lipschitz continuous ([37]), like in the case of nonexpansive mappings i.e.

$$\|Tx - Ty\| \leq L\|x - y\| \ \ \forall x, y \in K, (L > 0). \tag{4}$$

An element $x \in K$ is said to be a *fixed point* of T is $Tx = x$ and the set of fixed points of T is denoted by $F(T)$.

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The notion of strictly pseudocontractive operator in Hilbert spaces has been introduced and studied by Browder and Petryshyn [14], where the following result has been established.

Theorem 1

Let K be a bounded closed convex subset of a Hilbert space H and $T : K \to K$ be a k-strictly pseudocontractive mapping. Then for any given $x_0 \in K$ and any fixed number γ such that $1 - k < \gamma < 1$, the sequence $\{x_n\}_0^\infty$ given by

$$x_{n+1} = (1-\gamma)x_n + \gamma T x_n, n \geq 0, \tag{5}$$

converges weakly to some fixed point of T. If, in addition, T is demicompact, then $\{x_n\}_0^\infty$ converges strongly to some fixed point of T.

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Remark 2 [38]

A mapping $T : K \to H$ is called *demicompact* if it has the property that whenever $\{u_n\}$ is a bounded sequence in H and $\{Tu_n - u_n\}$ is strongly convergent, then there exists a subsequence $\{u_{n_k}\}$ of $\{u_n\}$ which is strongly convergent, where H is a Hilbert space and K a subset of H.

Definition 2 [9]

Let $(X, \|\cdot\|)$ be a linear normed space. A mapping $T : X \to X$ is called:

i) *enriched nonexpansive mapping* if there exists $b \in [0, \infty)$ such that

$$\|b(x-y) + Tx - Ty\| \leq (b+1)\|x-y\|, \forall x, y \in X. \tag{6}$$

To indicate the constant involved in (**??**) we shall also call T as a b-enriched nonexpansive mapping.

ii) *enriched strictly pseudocontractive mapping* if there exist $b \in [0, \infty)$ and $k < 1$ such that $\|b(x-y) + Tx - Ty\|^2 \leq$

$$\leq (b+1)^2\|x-y\|^2 + k\|x-y-(Tx-Ty)\|^2 \; \forall x, y \in X \tag{7}$$

We shall also call T as a (b, k)-enriched strictly pseudocontractive mapping.

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5. [11] Any b-enriched nonexpansive mapping is a (b, k)-enriched strictly pseudocontractive mapping for any $k < 1$.

Example 1

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3. [9] Let $X = \left[\frac{1}{2}, 2\right]$ be endowed with usual norm and $T : X \to X$ be defined by $Tx = \frac{1}{x}$, for all $x \in \left[\frac{1}{2}, 2\right]$. Then T is a $\frac{3}{2}$ - enriched nonexpansive mapping and $F(T) = \{1\}$.

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4. Let $k = \frac{1}{10}$ be a fixed number. Then $T : \left[\frac{1}{2}, 2\right] \to \left[\frac{1}{2}, 2\right]$ be defined by $Tx = \frac{1}{x}$, for all $x \in \left[\frac{1}{2}, 2\right]$ is a $\left(\frac{3}{2}, \frac{1}{10}\right)$ - enriched strictly pseudocontractive mapping and $F(T) = \{1\}$.

Definition 3 [14]

Let H be a Hilbert space and K a closed convex subset of H. A mapping $T : K \to K$ is called asymptotically regular (on K) if, for each $x \in K$,

$$\|T^{n+1}x - T^n x\| \to 0, \text{as } n \to \infty.$$

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Remark 4

Throughout this paper, we take H as a real Hilbert space with scalar product $\langle \cdot, \cdot \rangle$ and induced norm $\| \cdot \|$.

Weak convergence theorems for enriched strictly pseudocontractive mappings and enriched nonexpansive mappings by Krasnoselskij iteration

Theorem 2 [11]

Let K be a bounded closed convex subset of a Hilbert space H and $T : K \to K$ be a (b, k)-enriched strictly pseudocontractive mapping. Then $Fix(T) \neq \emptyset$ and, for any given $x_0 \in K$ and any fixed number γ, such that $0 < \gamma < 1 - k$, the Krasnoselskij iteration $\{x_n\}_{n=0}^{\infty}$ given by

$$x_{n+1} = (1 - \gamma)x_n + \gamma T x_n, n \geq 0, \tag{8}$$

converges weakly to a fixed point of T.

Weak convergence theorems for enriched strictly pseudocontractive mappings and enriched nonexpansive mappings by Krasnoselskij iteration

Theorem 3 [9]

Let K be a bounded closed convex subset of a Hilbert space H and $T : K \to K$ be an enriched nonexpansive operator with $Fix(T) = \{p\}$. Then, for any given $x_0 \in K$ and any fixed number γ, $0 < \gamma < 1$, the Krasnoselskij iteration $\{x_n\}_{n=0}^{\infty}$ given by

$$x_{n+1} = (1 - \gamma)x_n + \gamma Tx_n, n \geq 0, \tag{9}$$

converges weakly to p.

Strong convergence theorems for enriched strictly pseudocontractive mappings and enriched nonexpansive mappings by Krasnoselskij iteration

Theorem 4 [11]

Let K be a bounded closed convex subset of a Hilbert space H and $T : K \to K$ be a (b, k)-enriched strictly pseudocontractive and demicompact mapping. Then $Fix(T) \neq \emptyset$ and, for any given $x_0 \in K$ and any fixed number γ, such that $0 < \gamma < 1 - k$, the Krasnoselskij iteration $\{x_n\}_{n=0}^{\infty}$ given by

$$x_{n+1} = (1 - \gamma)x_n + \gamma T x_n, n \geq 0, \tag{10}$$

converges strongly to a fixed point of T.

Strong convergence theorems for enriched strictly pseudocontractive mappings and enriched nonexpansive mappings by Krasnoselskij iteration

Remark 5

Any k-strictly pseudocontractive mapping is a $(0, k)$-enriched strictly pseudocontractive mapping. Hence, the next corollary follows from Theorem 4 for $b = 0$, that is, for $\gamma = 1$.

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Corollary 1 [11]

Let K be a bounded closed convex subset of a Hilbert space H and $T : K \to K$ be a k-strictly pseudocontractive and demicompact operator. Then $Fix(T) \neq \emptyset$ and, for any given $x_0 \in K$ and any fixed number γ, such that $0 < \gamma < 1 - k$, the Krasnoselskij iteration $\{x_n\}_{n=0}^{\infty}$ given by

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Strong convergence theorems for enriched strictly pseudocontractive mappings and enriched nonexpansive mappings by Krasnoselskij iteration

Theorem 5 [9]

Let K be a bounded closed convex subset of a Hilbert space H and $T : K \to K$ be an enriched nonexpansive and demicompact mapping. Then the set $Fix(T)$ of fixed points of T is a nonempty convex set and there exists $\gamma \in (0, 1)$ such that, for any given $x_0 \in K$, the Krasnoselskij iteration $\{x_n\}_{n=0}^{\infty}$ given by

$$x_{n+1} = (1 - \gamma)x_n + \gamma T x_n, n \geq 0, \tag{12}$$

converges strongly to a fixed point of T.

Strong convergence theorems for enriched strictly pseudocontractive mappings and enriched nonexpansive mappings by Krasnoselskij iteration

Remark 6

Any nonexpansive mapping is a 0-enriched nonexpansive mapping. Hence, the next corollary follows from Theorem 5 for $b = 0$, that is, $\gamma = 1$.

Strong convergence theorems for enriched strictly pseudocontractive mappings and enriched nonexpansive mappings by Krasnoselskij iteration

Remark 6

Any nonexpansive mapping is a 0-enriched nonexpansive mapping. Hence, the next corollary follows from Theorem 5 for $b = 0$, that is, $\gamma = 1$.

Corollary 2 [14]

Let K be a bounded closed convex subset of a Hilbert space H and $T : K \to K$ be a nonexpansive and demicompact operator. Then the set $Fix(T)$ is a nonempty convex set and there exists $\gamma \in (0, 1)$ such that, for any given $x_0 \in K$, the Krasnoselskij iteration $\{x_n\}_{n=0}^{\infty}$ given by

$$x_{n+1} = (1 - \gamma)x_n + \gamma T x_n, n \geq 0, \tag{13}$$

converges strongly to a fixed point of T.

Numerical experiments

- For Table 1 and Table 2, let $Tx = \dfrac{1}{x}$, $x \in \left[\dfrac{1}{2}, 2\right]$. Then T is $\dfrac{3}{2}$ -enriched nonexpansive operator and $F(T) = \{1\}$. According to Remark 3, 5) any b-enriched nonexpansive mapping is a (b, k)-enriched strictly pseudocontractive mapping for any $k < 1$.

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- We choose in our examples, $k = \frac{1}{10}$. Hence, T is $\left(\frac{3}{2}, \frac{1}{10}\right)$ - enriched strictly pseudocontractive mapping.

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- We choose in our examples, $k = \dfrac{1}{10}$. Hence, T is $\left(\dfrac{3}{2}, \dfrac{1}{10}\right)$ - enriched strictly pseudocontractive mapping.

- It is easily seen that the Krasnoselskij iteration converges to $x^* = 1$, for $\gamma \in \{0.8, 0.7, 0.6, 0.5\}$ and $k = 0.1$. The starting value is $x_0 = \dfrac{1}{2}$ for Table 1 and $x_0 = 2$ for Table 2.

T. 1: Results for $\gamma \in \{0.8, 0.7, 0.6, 0.5\}$, $k = 0.1$, $x_0 = 0.5$

γ	0.8	0.7	0.6	0.5
k / n	0.1	0.1	0.1	0.1
0	0.5	0.5	0.5	0.5
1	1.7000	1.5500	1.4000	1.2500
2	0.8106	0.9166	0.9886	1.0250
3	1.1491	1.0387	1.0024	1.0003
4	0.9260	0.9855	0.9995	1
5	1.0491	1.0059	1.0001	1
6	0.9724	0.9977	1	1
7	1.0172	1.0009	1	1
8	0.9899	0.9996	1	1
9	1.0061	1.0002	1	1
10	0.9963	0.9999	1	1
11	1.0022	1	1	1
12	0.9987	1	1	1
N	18	10	5	3

T. 2: Results for $\gamma \in \{0.8, 0.7, 0.6, 0.5\}$, $k = 0.1$, $x_0 = 2$

γ / k / n	0.8	0.7	0.6	0.5
k	0.1	0.1	0.1	0.1
0	2	2	2	2
1	0.8000	0.9500	1.1000	1.2500
2	1.1600	1.0218	0.9855	1.0250
3	0.9217	0.9916	1.0030	1.0003
4	1.0523	1.0034	0.9994	1
5	0.9707	0.9986	1.0001	1
6	1.0183	1.0005	1	1
7	0.9893	0.9998	1	1
8	1.0065	1.0001	1	1
9	0.9961	1	1	1
10	1.0023	1	1	1
11	0.9986	1	1	1
12	1.0008	1	1	1
N	17	8	5	3

Numerical experiments

- For Table 3 and Table 4, let the same enriched nonexpansive operator $T,\ Tx = \dfrac{1}{x},\ x \in \left[\dfrac{1}{2}, 2\right]$, with $b = \dfrac{3}{2}$ and $F(T) = \{1\}$.

Numerical experiments

- For Table 3 and Table 4, let the same enriched nonexpansive operator $T,\ Tx = \frac{1}{x}$, $x \in \left[\frac{1}{2}, 2\right]$, with $b = \frac{3}{2}$ and $F(T) = \{1\}$.

- In the next tables, the Krasnoselskij iteration converges to $x^* = 1$, for $\gamma \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$. The starting value is $x_0 = \frac{1}{2}$ for Table 3 and $x_0 = 2$ for Table 4.

Table 3: Results for $\gamma \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$, $x_0 = 0.5$

n \ γ	0.1	0.2	0.3	0.4	0.5
0	0.5	0.5	0.5	0.5	0.5
1	0.6500	0.8000	0.9500	1.1000	1.2500
2	0.7388	0.8900	0.9808	1.0236	1.0250
3	0.8003	0.9367	0.9924	1.0049	1.0003
4	0.8452	0.9629	0.9970	1.0010	1
5	0.8790	0.9780	0.9988	1.0002	1
6	0.9049	0.9869	0.9995	1	1
7	0.9249	0.9922	0.9998	1	1
8	0.9405	0.9953	0.9999	1	1
9	0.9528	0.9972	1	1	1
10	0.9625	0.9983	1	1	1
11	0.9701	0.9990	1	1	1
12	0.9762	0.9994	1	1	1
13	0.9810	0.9996	1	1	1
N	39	16	8	5	3

Table 4: Results for $\gamma \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$, $x_0 = 2$

n \ γ	0.1	0.2	0.3	0.4	0.5
0	2	2	2	2	2
1	1.8500	1.700	1.5500	1.4000	1.2500
2	1.7191	1.4776	1.2785	1.1257	1.0250
3	1.6053	1.3175	1.1296	1.0308	1.0003
4	1.5071	1.2058	1.0663	1.0065	1
5	1.4227	1.1305	1.0234	1.0013	1
6	1.3507	1.0813	1.0095	1.0003	1
7	1.2897	1.0500	1.0038	1.0001	1
8	1.2383	1.0305	1.0015	1	1
9	1.1952	1.0185	1.0006	1	1
10	1.1593	1.0111	1.0002	1	1
11	1.1297	1.0067	1.0001	1	1
12	1.1052	1.0040	1	1	1
13	1.0852	1.0024	1	1	1
N	46	20	11	7	3

Remark 7

Let be the same enriched strictly pseudocontractive mapping with $b = \frac{3}{2}$ and $k = \frac{1}{10}$. For $\gamma = \frac{9}{10}$, the conditions of Theorem 2 are not satisfied. The Krasnoselskij iteration reduces to

$$x_{n+1} = \frac{1}{10}x_n + \frac{9}{10 \cdot x_n}$$

and, for $x_0 = 2$, we obtain

$$x_1 = 0.6500$$

$$x_2 = 1.4496$$

$$x_3 = 0.7658$$

$$x_4 = 1.2518$$

Numerical experiments

$$\vdots$$

$$x_{40} = 1.0001$$

$$x_{41} = 0.9999$$

$$x_{42} = 1$$

$$x_{43} = 1$$

$$x_{44} = 1$$

$$\vdots$$

We see that $\{x_n\}$ converge to $x^* = 1$. In this case, for $k = \dfrac{1}{10}$ and $\gamma = \dfrac{9}{10}$, the Krasnoseslkij iteration converges to 1 because the conditions of Theorem 3 are satisfied and T is an enriched nonexpansive mapping.

Conclusions

- In this paper, we studied the class of enriched strictly pseudocontractive and enriched nonexpansive mappings in the setting of a Hilbert space H. In order to approximate a fixed point of an enriched strictly pseudocontractive and an enriched nonexpansive mapping, we used the Krasnoselskij iteration. The focus of this paper is centered on weak convergence results (Theorem 2 and Theorem 3) and strong convergence results (Theorem 4 and Theorem 5).

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- These results extend some convergence theorems in [14] from strictly pseudocontractive mappings to enriched strictly pseudocontractive mappings and from nonexpansive mappings to enriched nonexpansive mappings.
- In the last part we presented some numerical experiments intended to illustrate the effectiveness of the Krasnoselskij iteration in the class of enriched strictly pseudocontractive mappings and enriched nonexpansive mappings.

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Thank you!