Weak snd Strong Convergence Theorems for Krasnoselskii Iterative algorithm in the class of enriched strictly pseudocontractive and enriched nonexpansive operators in Hilbert Spaces

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Abstract

In this paper, we present some results about the aproximation of fixed points of enriched strictly pseudocontractive and enriched nonexpansive operators. There are numerous works in this regard (for example [9], [10], [11] [14], [16], [35] and references to them). Of course, the bibliografical references are extensive and they are mentioned at the end of this paper. In order to approximate the fixed points of enriched strictly pseudocontractive and enriched nonexpansive mappings, we use the Krasnoselskii iterative algorithm for which we prove weak and strong convergence theorems.
Also, in this paper, we make a comparative study about some classical convergence theorems from the literature in the class of enriched strictly pseudocontractive and enriched nonexpansive mappings.

Keywords: *weak convergence, strong convergence, Krasnoselskii algorithm, enriched strictly pseudocontractive operator, enriched nonexpansive operator, Hilbert space.*

1. Introduction and Preliminaries

Definition 1.1. [9] Let K be a nonempty subset of a real normed linear space X. A mapping $T : K \to K$ is called:

i) *nonexpansive* if
$$\|Tx - Ty\| \le \|x - y\| \;\; \forall x, y \in K. \tag{1}$$

ii) *pseudocontractive* if
$$\|Tx - Ty\|^2 \le \|x - y\|^2 + \|x - y - (Tx - Ty)\| \;\; \forall x, y \in K. \tag{2}$$

iii) *strictly pseudocontractive* if there exist $k < 1$ such that
$$\|Tx - Ty\|^2 \le \|x - y\|^2 + k\|x - y - (Tx - Ty)\| \; \forall x, y \in K. \tag{3}$$

In this case, T is also called *k-strictly pseudocontractive*.

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Remark 1.2. It is easy to see that every nonexpansive mapping is strictly pseudocontractive and every strictly pseudocontractive mapping is pseudocontractive, but the reverses are not more true. Moreover, like in the case of nonexpansive mappings, any strictly pseudocontractive operator is Lipschitz continuous ([37]), i.e.

$$\|Tx - Ty\| \le L\|x - y\| \ \ \forall x, y \in K, (L > 0). \tag{4}$$

An element $x \in K$ is said to be a *fixed point* of T is $Tx = x$ and the set of fixed points of T is denoted by $F(T)$.

The notion of strictly pseudocontractive operator in Hilbert spaces has been introduced and studied by Browder and Petryshyn [14], where the following result has been established.

Theorem 1.3. *Let K be a bounded closed convex subset of a Hilbert space H and $T : K \to K$ be a k-strictly pseudocontractive mapping. Then for any given $x_0 \in K$ and any fixed number γ such that $1 - k < \gamma < 1$, the sequence $\{x_n\}_0^\infty$ given by*

$$x_{n+1} = (1 - \gamma)x_n + \gamma T x_n, n \ge 0, \tag{5}$$

converges weakly to some fixed point of T. If, in addition, T is demicompact, then $\{x_n\}_0^\infty$ converges strongly to some fixed point of T.

Remark 1.4. [38] We remember that a mapping $T : K \to H$ is called *demicompact* if it has the property that whenever $\{u_n\}$ is a bounded sequence in H and $\{Tu_n - u_n\}$ is strongly convergent, then there exists a subsequence $\{u_{n_k}\}$ of $\{u_n\}$ which is strongly convergent, where H is a Hilbert space and K a subset of H.

Definition 1.5. [9] Let $(X, \|\cdot\|)$ be a linear normed space. A mapping $T : X \to X$ is called:

i) *enriched nonexpansive mapping* if there exists $b \in [0, \infty)$ such that

$$\|b(x - y) + Tx - Ty\| \le (b + 1)\|x - y\|, \forall x, y \in X. \tag{6}$$

To indicate the constant involved in (**??**) we shall also call T as a b-enriched nonexpansive mapping.

ii) *enriched strictly pseudocontractive mapping* if there exist $b \in [0, \infty)$ and $k < 1$ such that

$$\|b(x - y) + Tx - Ty\|^2 \le$$
$$\le (b + 1)^2\|x - y\|^2 + k\|x - y - (Tx - Ty)\|^2 \ \forall x, y \in X \tag{7}$$

We shall also call T as a (b, k)-enriched strictly pseudocontractive mapping.

Remark 1.6. 1) [9] It is easy to see that any nonexpansive mapping T is a 0-enriched mapping, i.e., it satisfies (**??**) with $b = 0$.

2) [9] We note that, according to Theorem 12.1 in [25], in a Hilbert space any enriched nonexpansive mapping which is also firmly nonexpansive is nonexpansive. T is said to be *firmly nonexpansive* if

$$\|T(x) - T(y)\|^2 + \|(Id - T)(x) - (Id - T)(y)\|^2 \le \|x - y\|^2$$

$(x, y \in X)$.

3) [9] It is very important to note that, similar to the case of nonexpansive mappings, any enriched nonexpansive mapping is continuous.

4) [11] Any strictly pseudocontractive mapping T is a $(0, k)$-enriched strictly pseudocontractive mapping, i.e., it satisfies (**??**) with $b = 0$.

5) [11] Any b-enriched nonexpansive mapping is a (b, k)-enriched strictly pseudocontractive mapping for any $k < 1$.

Example 1.7. 1) $T : [0,4] \to [0,4]$, $Tx = 4 - x$, for all $x \in [0,4]$ is nonexpansive and T has a unique fixed point, $F(T) = \{2\}$.

2) If $T : [0,10] \to [0,10]$, $Tx = 2x - 10$, then T is not nonexpansive, because, for $x = 5$ and $y = 4$, then $\|Tx - Ty\| \leq \|x - y\| \Leftrightarrow 2 \leq 1$, which is false.

3) [9] Let $X = \left[\frac{1}{2}, 2\right]$ be endowed with usual norm and $T : X \to X$ be defined by $Tx = \frac{1}{x}$, for all $x \in \left[\frac{1}{2}, 2\right]$. Then T is a $\frac{3}{2}$ - enriched nonexpansive mapping and $F(T) = \{1\}$.

4) Let $k = \frac{1}{10}$ be a fixed number. Then $T : \left[\frac{1}{2}, 2\right] \to \left[\frac{1}{2}, 2\right]$ be defined by $Tx = \frac{1}{x}$, for all $x \in \left[\frac{1}{2}, 2\right]$ is a $\left(\frac{3}{2}, \frac{1}{10}\right)$ - enriched strictly pseudocontractive mapping and $F(T) = \{1\}$.

Definition 1.8. [14] Let H be a Hilbert space and K a closed convex subset of H. A mapping $T : K \to K$ is called asymptotically regular (on K) if, for each $x \in K$,

$$\|T^{n+1}x - T^n x\| \to 0, \text{as } n \to \infty.$$

Remark 1.9. Throughout this paper, we take H as a real Hilbert space with scalar product $\langle \cdot, \cdot \rangle$ and induced norm $\| \cdot \|$.

2. Weak convergence theorems for enriched strictly pseudocontractive mappings and enriched nonexpansive mappings by Krasnoselskij iteration

Theorem 2.1. *[11] Let K be a bounded closed convex subset of a Hilbert space H and $T : K \to K$ be a (b,k)-enriched strictly pseudocontractive mapping. Then $Fix(T) \neq \emptyset$ and, for any given $x_0 \in K$ and any fixed number γ, such that $0 < \gamma < 1 - k$, the Krasnoselskij iteration $\{x_n\}_{n=0}^{\infty}$ given by*

$$x_{n+1} = (1 - \gamma)x_n + \gamma T x_n, n \geq 0, \tag{8}$$

converges weakly to a fixed point of T.

Theorem 2.2. *[9] Let K be a bounded closed convex subset of a Hilbert space H and $T : K \to K$ be an enriched nonexpansive operator with $Fix(T) = \{p\}$. Then, for any given $x_0 \in K$ and any fixed number γ, $0 < \gamma < 1$, the Krasnoselskij iteration $\{x_n\}_{n=0}^{\infty}$ given by*

$$x_{n+1} = (1 - \gamma)x_n + \gamma T x_n, n \geq 0, \tag{9}$$

converges weakly to p.

3. Strong convergence theorems for enriched strictly pseudocontractive mappings and enriched nonexpansive mappings by Krasnoselskij iteration

Theorem 3.1. *[11] Let K be a bounded closed convex subset of a Hilbert space H and $T : K \to K$ be a (b,k)-enriched strictly pseudocontractive and demicompact mapping. Then $Fix(T) \neq \emptyset$ and, for any given $x_0 \in K$ and any fixed number γ, such that $0 < \gamma < 1 - k$, the Krasnoselskij iteration $\{x_n\}_{n=0}^{\infty}$ given by*

$$x_{n+1} = (1 - \gamma)x_n + \gamma T x_n, n \geq 0, \tag{10}$$

converges strongly to a fixed point of T.

Remark 3.2. Any k-strictly pseudocontractive mapping is a $(0,k)$-enriched strictly pseudocontractive mapping. Hence, the next corollary follows from Theorem **??** for $b = 0$, that is, for $\gamma = 1$.

Corollary 3.3. *[11] Let K be a bounded closed convex subset of a Hilbert space H and $T : K \to K$ be a k-strictly pseudocontractive and demicompact operator. Then $Fix(T) \neq \emptyset$ and, for any given $x_0 \in K$ and any fixed number γ, such that $0 < \gamma < 1 - k$, the Krasnoselskij iteration $\{x_n\}_{n=0}^{\infty}$ given by*

$$x_{n+1} = (1 - \gamma)x_n + \gamma T x_n, n \geq 0, \tag{11}$$

converges strongly to a fixed point of T.

Theorem 3.4. *[9] Let K be a bounded closed convex subset of a Hilbert space H and $T : K \to K$ be an enriched nonexpansive and demicompact mapping. Then the set $Fix(T)$ of fixed points of T is a nonempty convex set and there exists $\gamma \in (0, 1)$ such that, for any given $x_0 \in K$, the Krasnoselskij iteration $\{x_n\}_{n=0}^{\infty}$ given by*

$$x_{n+1} = (1 - \gamma)x_n + \gamma T x_n, n \geq 0, \tag{12}$$

converges strongly to a fixed point of T.

Remark 3.5. Any nonexpansive mapping is a 0-enriched nonexpansive mapping. Hence, the next corollary follows from Theorem **??** for $b = 0$, that is, $\gamma = 1$.

Corollary 3.6. *[14] Let K be a bounded closed convex subset of a Hilbert space H and $T : K \to K$ be an nonexpansive and demicompact operator. Then the set $Fix(T)$ of fixed points of T is a nonempty convex set and there exists $\gamma \in (0, 1)$ such that, for any given $x_0 \in K$, the Krasnoselskij iteration $\{x_n\}_{n=0}^{\infty}$ given by*

$$x_{n+1} = (1 - \gamma)x_n + \gamma T x_n, n \geq 0, \tag{13}$$

converges strongly to a fixed point of T.

4. Numerical experiments

We present some numerical experiments intended to illustrate the effectiveness of the Krasnoselskii iteration in the class of enriched strictly pseudocontractive mappings and enriched nonexpansive mappings.

For Table 1 and Table 2, let $Tx = \frac{1}{x}$, $x \in \left[\frac{1}{2}, 2\right]$. Then T is $\frac{3}{2}$ - enriched nonexpansive operator and $F(T) = \{1\}$. According to Remark **??**, 5) any b-enriched nonexpansive mapping is a (b, k)-enriched strictly pseudocontractive mapping for any $k < 1$. We choose in our example, $k = \frac{1}{10}$. Hence, T is $\left(\frac{3}{2}, \frac{1}{10}\right)$ - enriched strictly pseudocontractive mapping.
It is easily seen that the Krasnoselskii iteration converges to $x^* = 1$, for $\gamma \in \{0.8, 0.7, 0.6, 0.5\}$ and $k = 0.1$. The starting value is $x_0 = \frac{1}{2}$ for Table 1 and $x_0 = 2$ for Table 2. The numerical experiments illustrate the convergence of the Krasnoselskii iteration. N denotes the number of iterations needed to reach the exact solution with four exact digits. Note also the fact that, for fixed $k = 0.1$ and for high values of γ, the Krasnoselskii iteration converges slowly, while for small values of γ, it converges faster.

Table 1: Results of the numerical experiments for $\gamma \in \{0.8, 0.7, 0.6, 0.5\}$, $k = 0.1$ and $x_0 = \frac{1}{2}$.

γ	0.8	0.7	0.6	0.5
k	0.1	0.1	0.1	0.1
n				
0	0.5	0.5	0.5	0.5
1	1.7000	1.5500	1.4000	1.2500
2	0.8106	0.9166	0.9886	1.0250
3	1.1491	1.0387	1.0024	1.0003
4	0.9260	0.9855	0.9995	1
5	1.0491	1.0059	1.0001	1
6	0.9724	0.9977	1	1
7	1.0172	1.0009	1	1
8	0.9899	0.9996	1	1
9	1.0061	1.0002	1	1
10	0.9963	0.9999	1	1
11	1.0022	1	1	1
12	0.9987	1	1	1
N	18	10	5	3

Table 2: Results of the numerical experiments for $\gamma \in \{0.8, 0.7, 0.6, 0.5\}$, $k = 0.1$ and $x_0 = 2$.

γ	0.8	0.7	0.6	0.5
k	0.1	0.1	0.1	0.1
n				
0	2	2	2	2
1	0.8000	0.9500	1.1000	1.2500
2	1.1600	1.0218	0.9855	1.0250
3	0.9217	0.9916	1.0030	1.0003
4	1.0523	1.0034	0.9994	1
5	0.9707	0.9986	1.0001	1
6	1.0183	1.0005	1	1
7	0.9893	0.9998	1	1
8	1.0065	1.0001	1	1
9	0.9961	1	1	1
10	1.0023	1	1	1
11	0.9986	1	1	1
12	1.0008	1	1	1
N	17	8	5	3

For Table 3 and Table 4, let the same enriched nonexpansive operator T, $Tx = \frac{1}{x}$, $x \in \left[\frac{1}{2}, 2\right]$, with $b = \frac{3}{2}$ and $F(T) = \{1\}$.

In the next table, the Krasnoselskii iteration converges to $x^* = 1$, for $\gamma \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$. The starting value is $x_0 = \frac{1}{2}$ for Table 3 and $x_0 = 2$ for Table 4. The numerical experiments illustrate the convergence of the Krasnoselskii iteration. N denotes the number of iterations needed to reach the exact solution with four exact digits. Note also the fact that, for small values of γ, the Krasnoselskii iteration converges slowly, while for high values of γ, it converges faster.

Table 3: Results of the numerical experiments for $\gamma \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$ and $x_0 = \frac{1}{2}$.

γ / n	0.1	0.2	0.3	0.4	0.5
0	0.5	0.5	0.5	0.5	0.5
1	0.6500	0.8000	0.9500	1.1000	1.2500
2	0.7388	0.8900	0.9808	1.0236	1.0250
3	0.8003	0.9367	0.9924	1.0049	1.0003
4	0.8452	0.9629	0.9970	1.0010	1
5	0.8790	0.9780	0.9988	1.0002	1
6	0.9049	0.9869	0.9995	1	1
7	0.9249	0.9922	0.9998	1	1
8	0.9405	0.9953	0.9999	1	1
9	0.9528	0.9972	1	1	1
10	0.9625	0.9983	1	1	1
11	0.9701	0.9990	1	1	1
12	0.9762	0.9994	1	1	1
13	0.9810	0.9996	1	1	1
N	39	16	8	5	3

Table 4: Results of the numerical experiments for $\gamma \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$ and $x_0 = 2$.

γ / n	0.1	0.2	0.3	0.4	0.5
0	2	2	2	2	2
1	1.8500	1.700	1.5500	1.4000	1.2500
2	1.7191	1.4776	1.2785	1.1257	1.0250
3	1.6053	1.3175	1.1296	1.0308	1.0003
4	1.5071	1.2058	1.0663	1.0065	1
5	1.4227	1.1305	1.0234	1.0013	1
6	1.3507	1.0813	1.0095	1.0003	1
7	1.2897	1.0500	1.0038	1.0001	1
8	1.2383	1.0305	1.0015	1	1
9	1.1952	1.0185	1.0006	1	1
10	1.1593	1.0111	1.0002	1	1
11	1.1297	1.0067	1.0001	1	1
12	1.1052	1.0040	1	1	1
13	1.0852	1.0024	1	1	1
N	46	20	11	7	3

Remark 4.1. Let be the same enriched strictly pseudocontractive mapping with $b = \frac{3}{2}$ and $k = \frac{1}{10}$. For $\gamma = \frac{9}{10}$, the conditions of Theorem **??** are not satisfied. The Krasnoselskii iteration reduces to

$$x_{n+1} = \frac{1}{10} x_n + \frac{9}{10 \cdot x_n}$$

and, for $x_0 = 2$, we obtain

$$x_1 = 0.6500$$

$$x_2 = 1.4496$$

$$x_3 = 0.7658$$

$$x_4 = 1.2518$$
$$\vdots$$
$$x_{40} = 1.0001$$
$$x_{41} = 0.9999$$
$$x_{42} = 1$$
$$x_{43} = 1$$
$$x_{44} = 1$$
$$\vdots$$

We see that $\{x_n\}$ converge to $x^* = 1$. In this case, for $k = \frac{1}{10}$ and $\gamma = \frac{9}{10}$, the Krasnoseslkii iteration converges to 1 because the conditions of Theorem **??** are satisfied and T is an enriched nonexpansive mapping.

5. Final remarks

In this paper, we studied the class of enriched strictly pseudocontractive and enriched nonexpansive mappings in the setting of a Hilbert space H. In order to approximate a fixed point of an enriched strictly pseudocontractive and an enriched nonexpansive mapping, we used the Krasnoselskij iteration. The focus of this paper is centered on weak convergence results (Theorem **??** and Theorem **??**) and strong convergence results (Theorem **??** and Theorem **??**).
These results extend some convergence theorems in [14] from strictly pseudocontractive mappings to enriched strictly pseudocontractive mappings and from nonexpansive mappings to enriched nonexpansive mappings.
In the last part we presented some numerical experiments intended to illustrate the effectiveness of the Krasnoselskii iteration in the class of enriched strictly pseudocontractive mappings and enriched nonexpansive mappings.

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