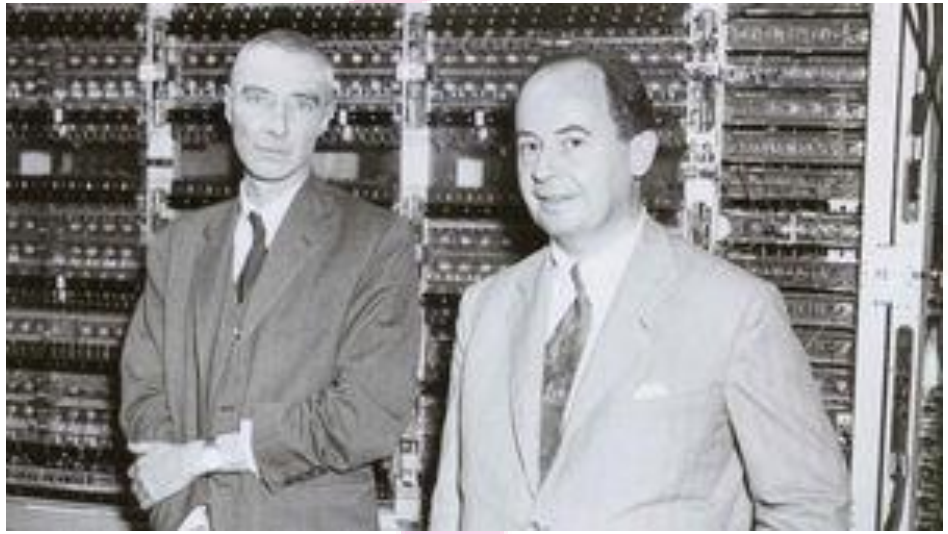
XGEN R&D

Game Theory

Flavia Tămaş

WHAT is Game Theory?

Game Theory facilitates understanding situations involving interactions among decision-makers. The initial development of game theory is attributed to John von Neumann, a Hungarian mathematician, and his colleague Oskar Morgenstern, a German economist at Princeton University. Developed to understand and predict human behaviour in competitive situations, it has now found applications in various fields, including political science, biology, and computer science.



Picture 1. John von Neumman and Oskar Morgenstern. Britannica, Inc.

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Nash Equilibrium

Nash Equilibrium indicates a state where no player can improve their payoff by altering their strategy independently.

Put simply, Sarah and John are in Nash equilibrium if Sarah is making the most optimal choice possible, considering John's decision, and John is making the most optimal choice he can, considering Sarah's decision.

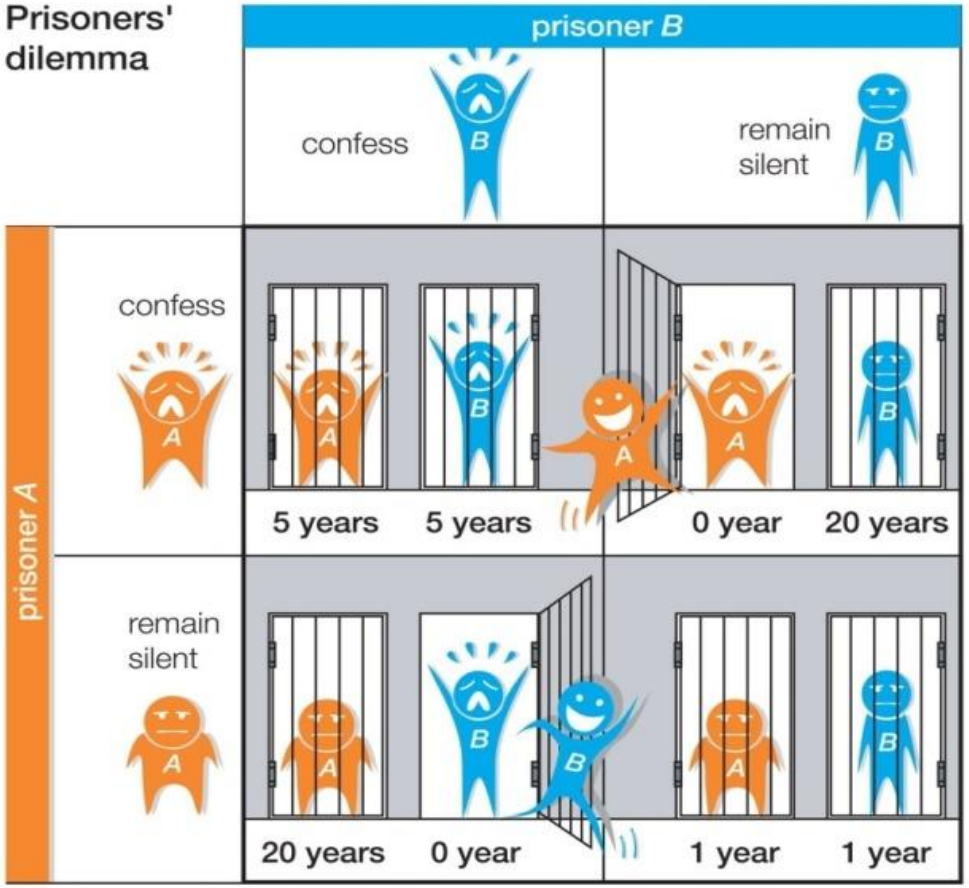
Nevertheless, the Nash equilibrium does not always ensure the highest possible payoffs for all participants. In numerous instances, all players could enhance their payoffs if they could mutually decide on strategies diverging from the Nash equilibrium.

Priosner’s Dilemma

Two individuals are arrested for a crime. Sufficient evidence exists to incriminate both individuals for a minor transgression, yet lacking adequate proof to convict either for a major crime, unless one chooses to inform against the other. If they opt for silence, each will be found guilty of the minor offence, resulting in a one-year prison term. However, if one individual decides to inform while the other remains silent, the informant will be released and utilized as a witness against their co-accused, who will then endure a twenty-year imprisonment. If both choose to inform, a five-year prison sentence awaits each party. As neither prisoner possesses certainty regarding the other's decision, **how will the prisoners proceed?**

Suspect 1’s ordering, from best to worst, is (Confess, Quiet), (Quiet, Quiet), (Confess, Confess), (Quiet, Confess). Suspect 2’s ordering is (Quiet, Confess), (Quiet, Quiet), (Confess, Confess), (Confess, Quiet).

The Nash Equilibrium in this case occurs when both players choose to confess, resulting in a five-year prison sentence for each player. This is because each player, when considering the other's possible actions, will find that confessing is the dominant strategy. Regardless of what the other player chooses, confessing provides the best possible outcome for each player individually.



Picture 2. Prisoner’s Dilemma. Encyclopedia Britannica, Inc.

Solved Problem

The two leading bookstores in the town of Baia Mare, "Readers' Haven" and "Bookworm's Delight," are considering implementing a loyalty card program to attract more customers. They have the option to either introduce a loyalty card that offers a 10% discount on every fifth purchase or refrain from such a program. The payoff matrix below illustrates the daily profits for each combination of decisions:

(Flavia Tămaş)

Reader's Heaven

Bookworm's Delight

	Loyalty Card	No Loyalty Card
Loyalty Card	$800, $700	$600, $650
No Loyalty Card	$750, $820	$680, $710

a) Determine whether implementing the loyalty card program is a dominant strategy for either bookstore based on the given values.

Answer:

- It is in the best interest of Reader's Heaven to implement a loyalty card, indicating a dominant strategy for them.
- It doesn't have a dominant strategy and should adjust its decision based on Reader's Heaven's actions.

Reader's Heaven

Bookworm's Delight

	Loyalty Card	No Loyalty Card
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b) Identify whether there is a Nash equilibrium in this scenario. If so, calculate the profits for each bookstore at this equilibrium.

- Since Reader's Haven has a dominant strategy to implement the loyalty card, the Nash equilibrium would be when Reader's Haven implements the card and Bookworm's Delight as well. At this equilibrium, Reader's Haven's profit would be $700, and Bookworm's Delight's profit would be $800.

Reader’s Heaven

Bookworm’s Delight

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c) If "Readers' Haven" and "Bookworm's Delight" both decide to implement the loyalty card program and offer a 10% discount on every second purchase, calculate how many purchases a customer needs to make for the total amount saved through the loyalty card to exceed the cost of the loyalty card($30), if a book has the price of $15?

The loyalty card program offers a 10% discount on every second purchase, and each book has a price of $15, the savings per two purchases would be $15 × 10% = $1,5.

$1,5.....2 purchases

$30..... x purchases

x = 40 purchases

- Therefore, a customer needs to make 40 purchases to exceed the cost of the loyalty card.

Practice Problems

1. Harry and Sally want to go out in the afternoon. There are two shops nearby, Yidiandian and Starbucks. The following table shows their payoffs:

a) Suppose Harry and Sally decide simultaneously, what is the Nash equilibrium.

b) Suppose Sally moves first and leaves Harry a text message where he can find her, and then Harry moves, what will be the new equilibrium?

Hint: Revise “The Battle of Sexes”

		Harry	
		Yi diandian	Starbucks
Sally	Yidiandian	2, 1	0, 0
	Starbucks	0, 0	1, 2

Picture 3. Problem 1 scenario. Studocu.

2. Mariam Industries and Luk, Ltd. are the only two firms in an oligopoly market, and each firm’s objective is to maximize its own profit. Each firm can maintain its current amount of output or increase its output. The payoffs shown in this table represent the change in each firm’s profit in each possible outcome. The first entry represents Luk Ltd.'s payoff and the second entry represents Mariam Industries' payoff. What is the change in industry profits in the Nash equilibrium of this game?

		Mariam Industries	
		maintain output	increase output
Luk, Ltd.	maintain output	$0 , $0	$0 , − $10
	inrease output	−$10 , $0	−$2 , − $2

Picture 7. Problem 2 scenario. Khan Academy.

Conclusion

Game Theory proves to be a versatile and powerful tool, extending its influence far beyond economics into realms such as political science and biology. The concept of Nash equilibrium, highlights the stability of strategies in the face of opponents' choices, guiding decision-making in diverse real-world scenarios. With applications across various fields, Game Theory offers valuable insights into the complexities of decision-making, providing a predictive framework for behaviors in competitive settings.

Joke

Why did the game theorist bring a ladder to the poker game?

Because he heard the stakes were high and wanted to reach the Nash Equilibrium! ☺

Thank you for your attention!

Biography

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